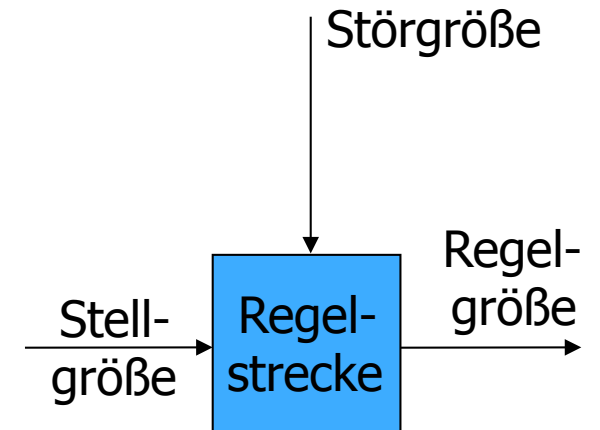
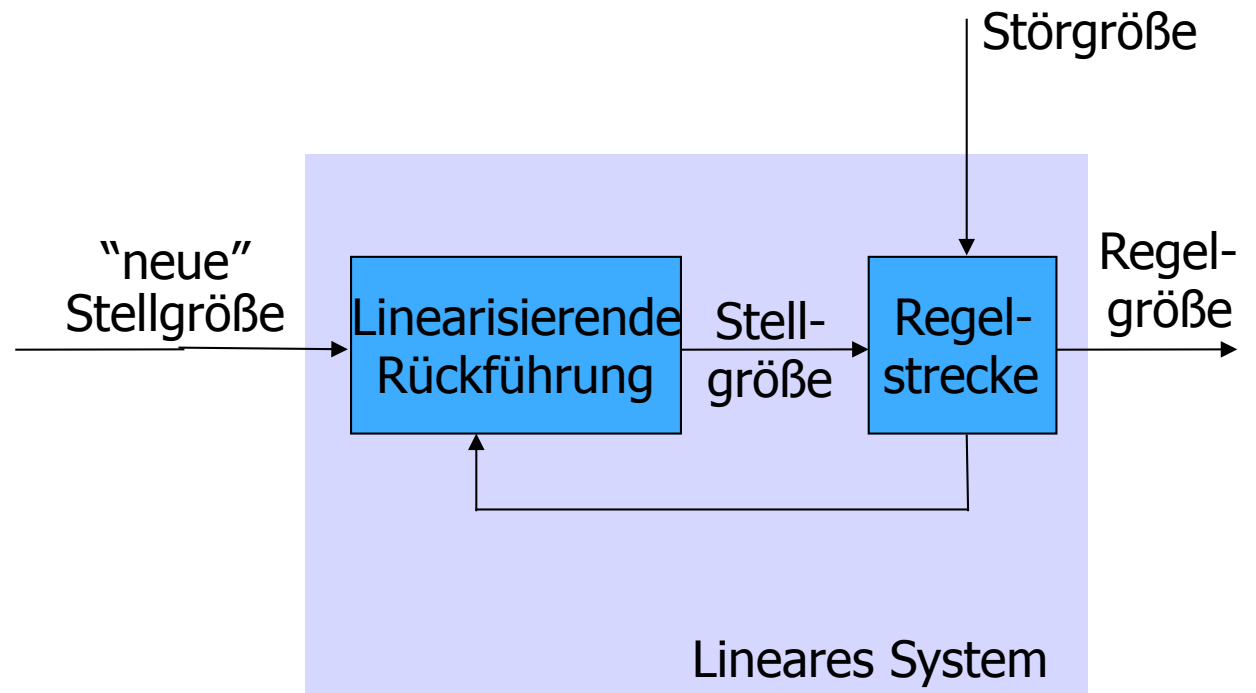


Idee: 2-Schritt Reglerentwurf



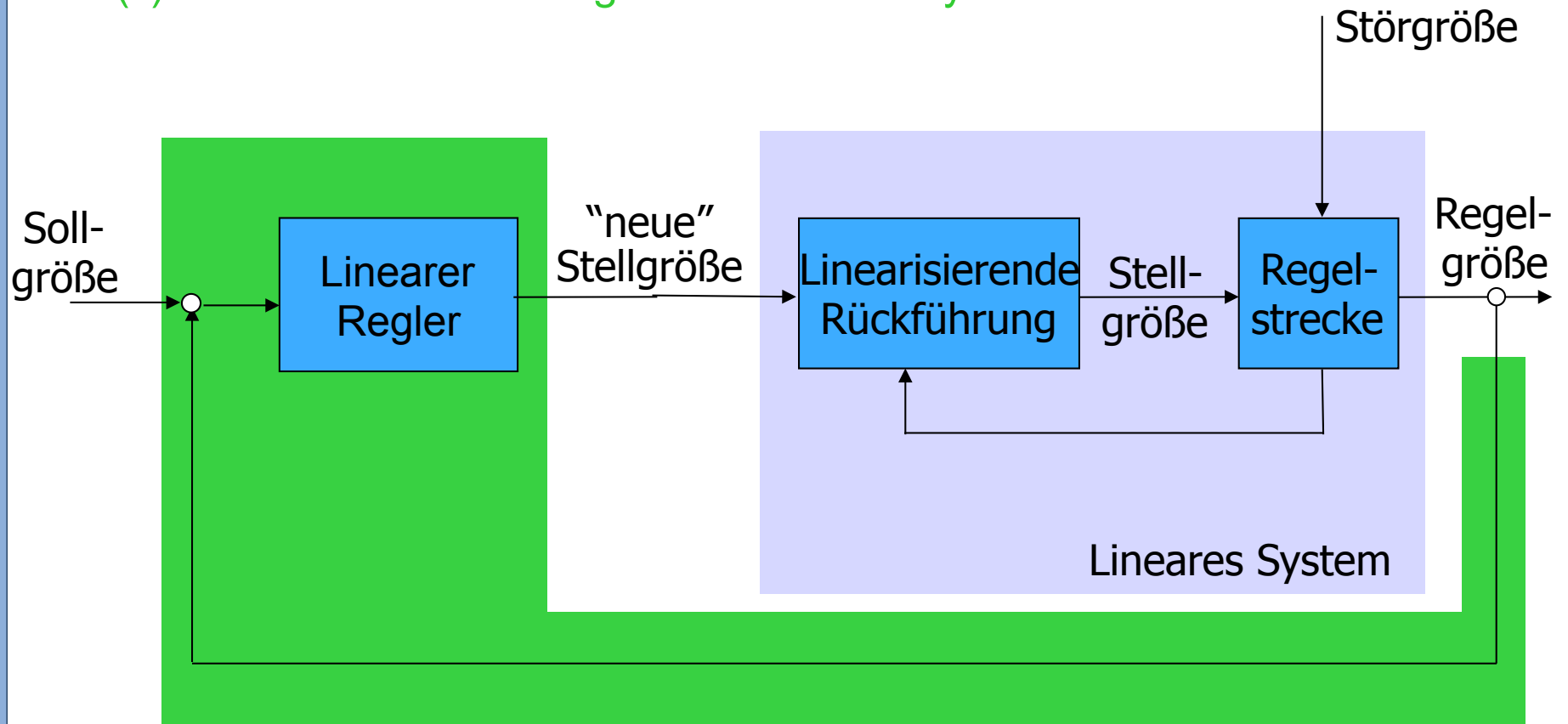
Idee: 2-Schritt Reglerentwurf

- (i) Berechne Rückführung, so dass E/A-Verhalten linear



Idee: 2-Schritt Reglerentwurf

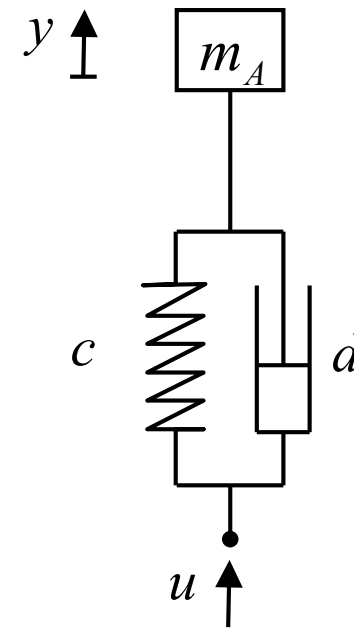
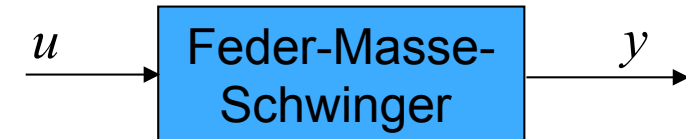
- (i) Berechne Rückführung, so dass E/A-Verhalten linear
- (ii) Entwerfe linearen Regler für lineares System



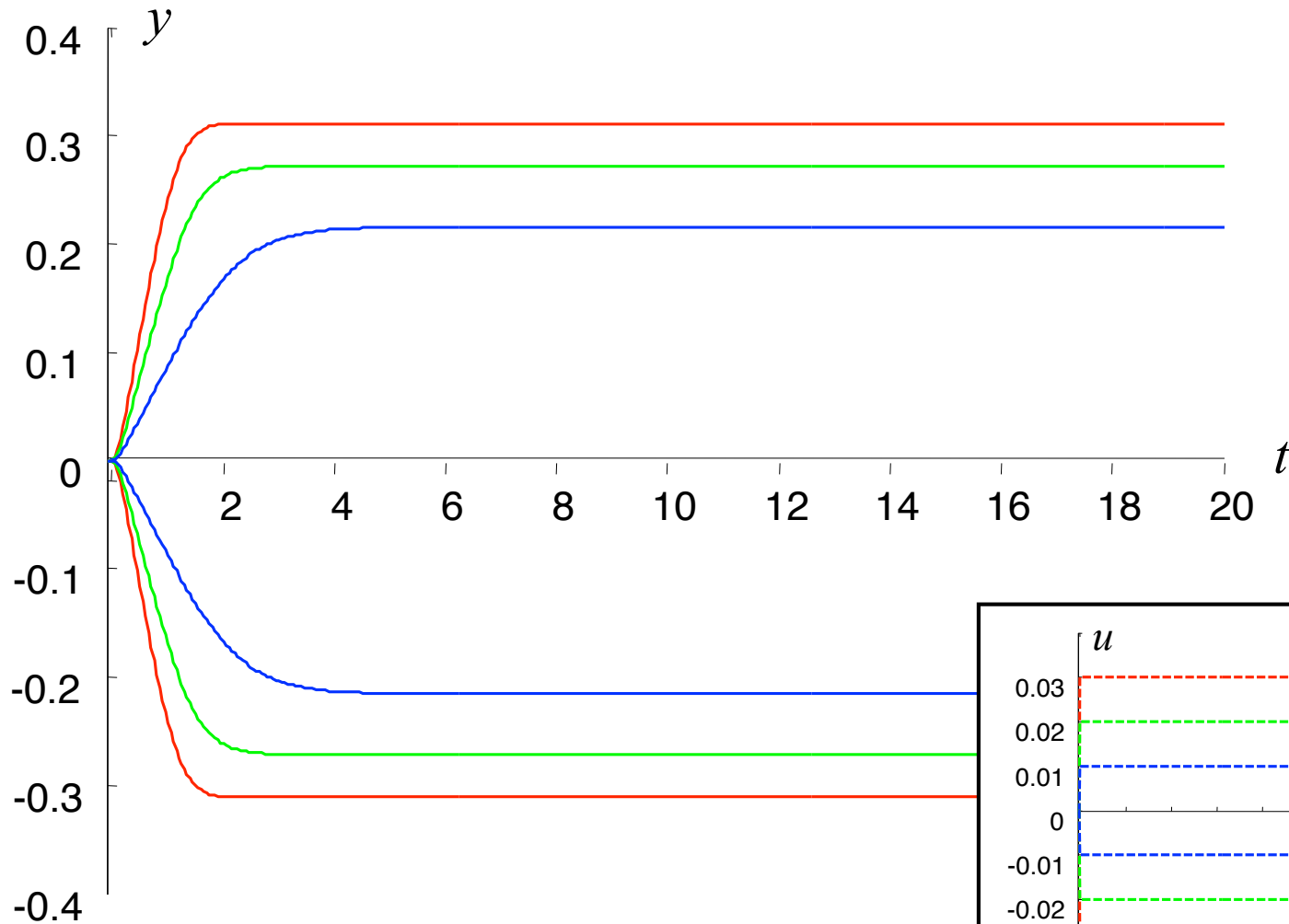
Beispiel: nichtlin. Feder-Masse-Schwinger

Systemgleichung:

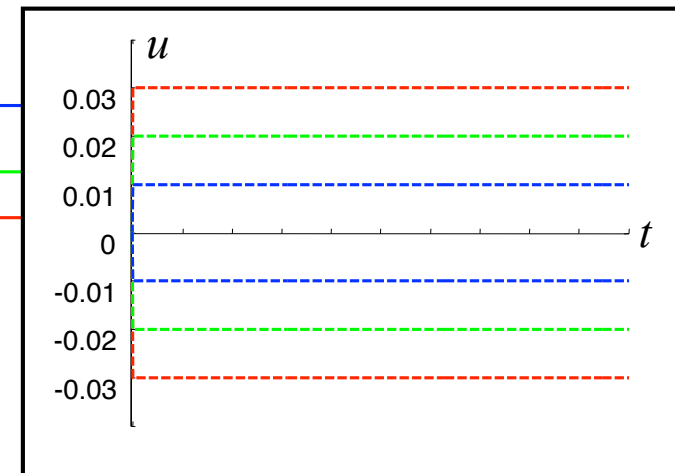
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{c}{m} x_1^3 - \frac{d}{m} x_2 + u \end{bmatrix}$$
$$y = x_1$$



Sprungantworten des nichtlinearen Systems



Homogenitätsprinzip verletzt!



Beispiel: nichtlin. Feder-Masse-Schwinger

Systemgleichung:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{c}{m} x_1^3 - \frac{d}{m} x_2 + u \end{bmatrix}$$

$$y = x_1$$

Linearisierende Rückführung

$$\dot{x}_2 = -\frac{c}{m} x_1^3 - \frac{d}{m} x_2 + u = v!$$

$$\Rightarrow u = v + \frac{c}{m} x_1^3 + \frac{d}{m} x_2$$

$$\Rightarrow \ddot{y} = v$$

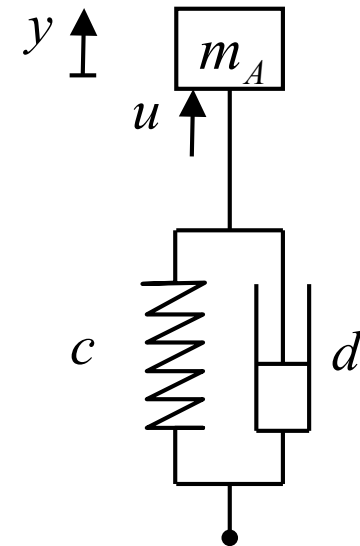
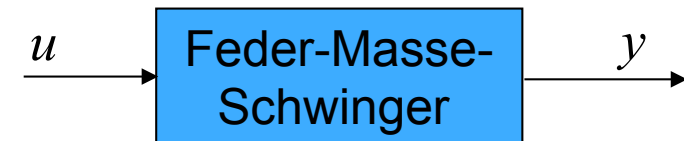
$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ v \end{bmatrix}$$

Lineares Übertragungsverhalten

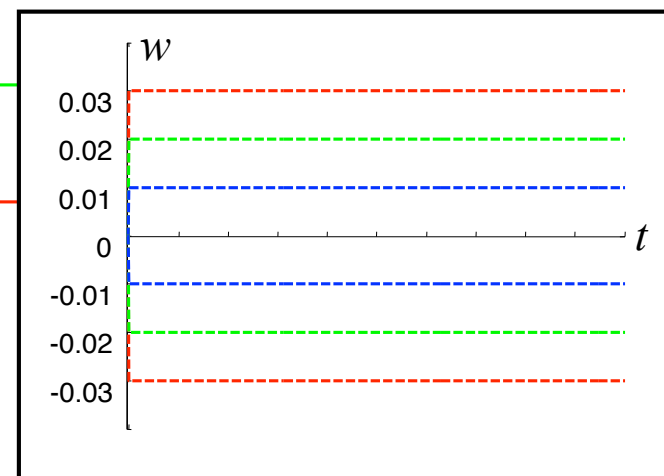
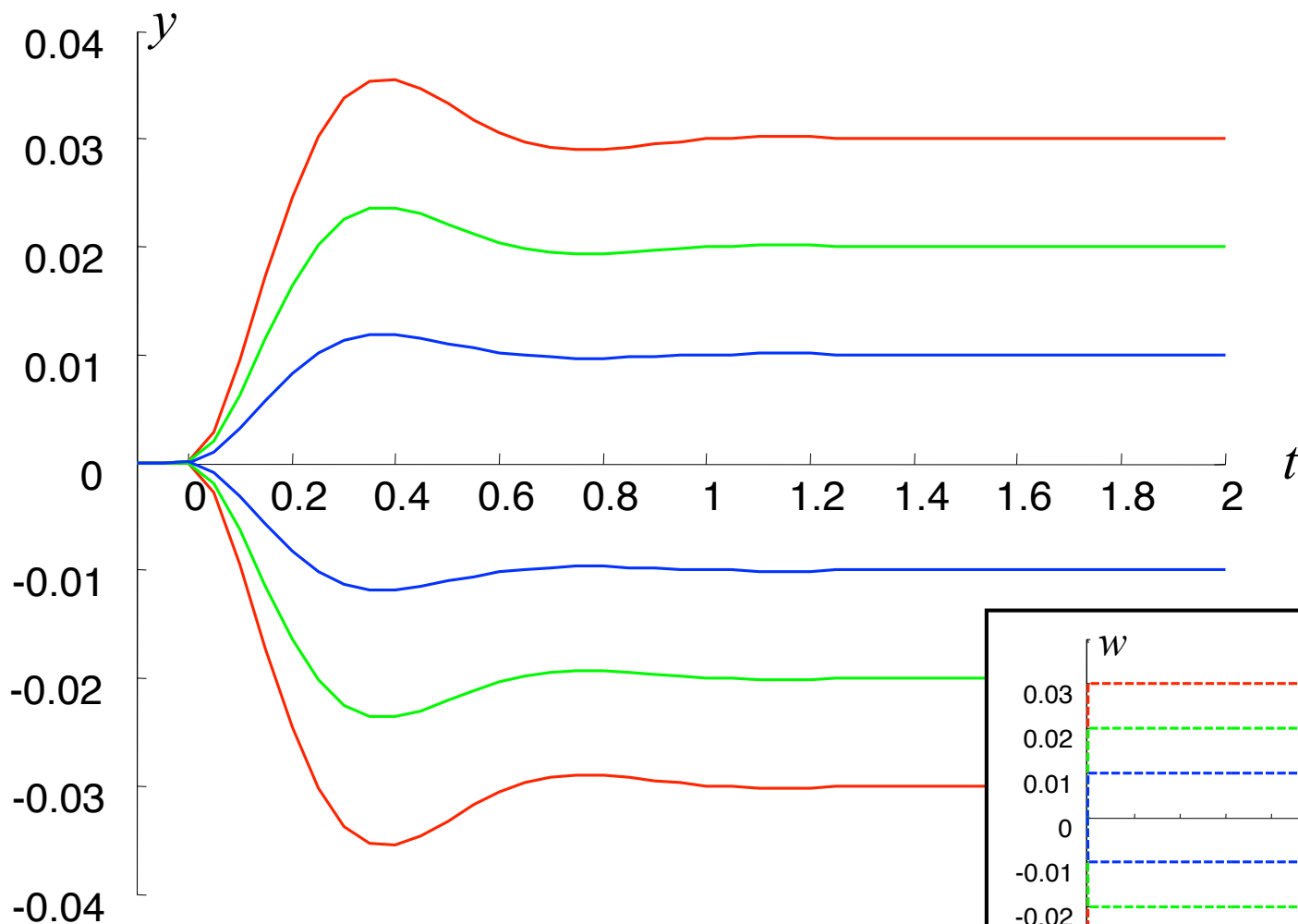
Lineare Zustandsgleichungen

Linearer Regler, z.B. Zustandsrückführung

$$v = -\frac{c}{m} x_1 - \frac{d}{m} x_2 + w$$

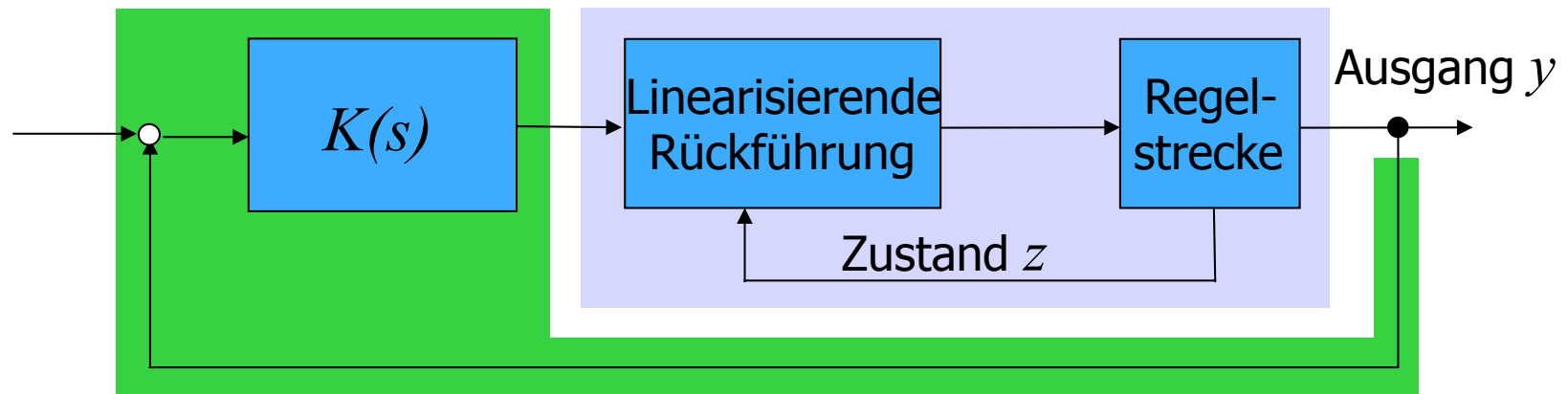


Sprungantworten des linearisierten Systems

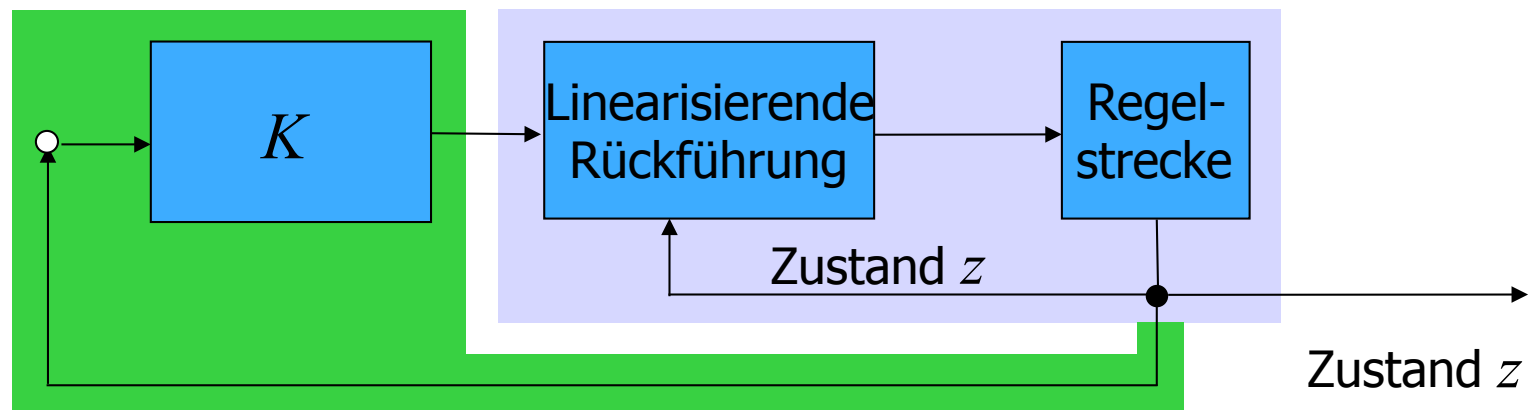


Exakte Linearisierung: Zwei Aufgabenstellungen

1. Exakte Ein-/Ausgangslinearisierung



2. Exakte Zustandslinearisierung:



Exakte Ein-/Ausganglinearisierung

$$\begin{cases} \dot{x} = f(x) + g(x)u, & x \in \mathbb{R}^n, u, y \in \mathbb{R}^m \\ y = h(x) \end{cases}$$

Zustandsrückführung $u = \alpha(x) + \beta(x)v$

$y = G(s)v$

Ziel: Entwerfe Regler

$$v(s) = -K(s)y(s)$$

so dass geschlossener Kreis

1. asymptotisch stabil ist
2. gewünschte Regelgüte besitzt

Exakte Zustandslinearisierung:

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m$$

Zustandsrückführung $u = \alpha(x) + \beta(x)v$

und Zustandstransformation $z = T(x)$

$$\dot{z} = Az + Bv, \quad z \in \mathbb{R}^n, \quad v \in \mathbb{R}^m$$

Hauptziel: Stabilisierung mit $v = -Kz$

Beispiel: Exakte Zustandslinearisierung eines Rührkesselreaktors



Reaktionsschema



Zustandsraumdarstellung

$$\dot{x}_A = \frac{q}{V} (c_{A_0} - c_A) - k_1(T) c_A^2$$

$$\dot{x}_B = -\frac{q}{V} c_B + k_1(T) c_A^2 - k_2(T) c_B$$

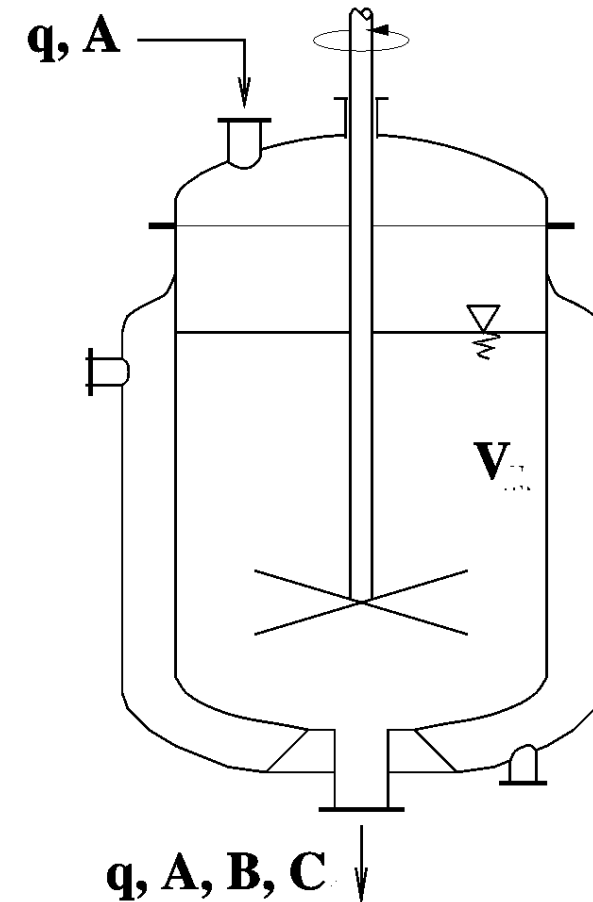
$$\dot{T} = \frac{q}{V} (T_0 - T) - \frac{1}{\rho C_P} \left(k_1(T) c_A \Delta H_{R_{AB}} + k_2(T) c_B \Delta H_{R_{BC}} \right) + \frac{k_w A_R}{\rho C_P V_R} (T_C - T)$$

mit

$$k_1(T) = k_{10} e^{-\frac{E_1}{RT}}$$

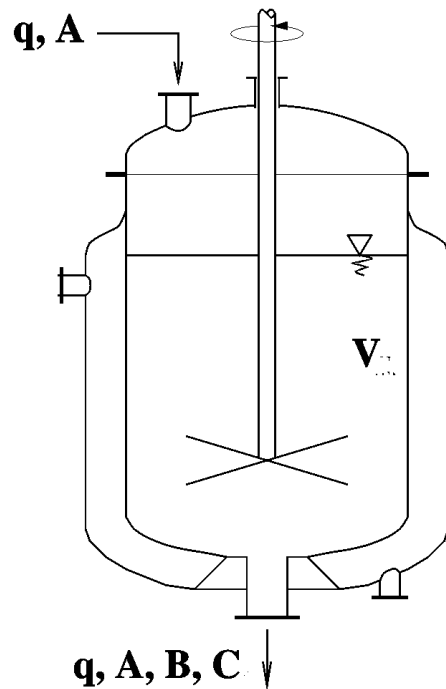
$$k_2(T) = k_{20} e^{-\frac{E_2}{RT}}$$

nur zustandslinearisierbar für $E_1 = E_2$!



Beispiel: Exakte Zustandslinearisierung eines Rührkesselreaktors

ist



$$u = \alpha(x) + \beta(x) \cdot v$$

mit $k_0 = \int_{-\infty}^x \frac{e^t}{t} dt$

```

OCONST*{-(H2*K20*X2*EXP(-E/(R*X3))+H1*K10*X1**2*EXP(-E/
1  (R*X3))+K0*(T1-X3))*{(K0*K20*EXP(K20/(K10*X1)))/(K10*X1)-CAI*K0*
2  K20*EXP(K20/(K10*X1)))/(K10*X1**2)-K0*EXP(K20/(K10*X1)))*{E*K10*
3  X1**2*EXP(-E/(R*X3))/(R*X3**2)-E*K20*X2*EXP(-E/(R*X3))/(R*X3**2
4  )-E*K10*X1**2*(-K0*K20**2*EXP(K20/(K10*X1))*X2/(K10**2*X1**3)+
5  2*CAI*K0*K20*EXP(K20/(K10*X1))*X2/(K10*X1**3)+CAI*K0*K20**2*EXP
6  (K20/(K10*X1))*X2/(K10**2*X1**4)+K0*K20*EXP(K20/(K10*X1))/(K10*
7  X1)-CAI*K0*K20*EXP(K20/(K10*X1))/(K10*X1**2)-K0*EXP(K20/(K10*X1
8  ))*EXP(-E/(R*X3))/(R*X3**2))-A2*((K0*K20*EXP(K20/(K10*X1))/(K1
9  0*X1)-CAI*K0*K20*EXP(K20/(K10*X1))/(K10*X1**2)-K0*EXP(K20/(K10*
10 X1))*{-K20*X2*EXP(-E/(R*X3))+K10*X1**2*EXP(-E/(R*X3))-K0*X2)+(-
11 -K0*K20**2*EXP(K20/(K10*X1))*X2/(K10**2*X1**3)+2*CAI*K0*K20*EXP
12 (K20/(K10*X1))*X2/(K10*X1**3)+CAI*K0*K20**2*EXP(K20/(K10*X1))*X
13 2/(K10**2*X1**4)+K0*K20*EXP(K20/(K10*X1))/(K10*X1)-CAI*K0*K20*E
14 XP(K20/(K10*X1))/(K10*X1**2)-K0*EXP(K20/(K10*X1))*{(K0*(CAI-X1)
15 -K10*X1**2*EXP(-B/(R*X3)))-{(K0*(CAI-X1)-K10*X1**2*EXP(-E/(R*X3
16 )))*{-K0*K20**2*EXP(K20/(K10*X1))/(K10**2*X1**3)+2*CAI*K0*K20*
17 EXP(K20/(K10*X1))/(K10*X1**3)+CAI*K0*K20**2*EXP(K20/(K10*X1))/(
18 K10**2*X1**4))*{-K20*X2*EXP(-E/(R*X3))+K10*X1**2*EXP(-E/(R*X3))
19 -K0*X2)+(3*K0*K20**2*EXP(K20/(K10*X1))*X2/(K10**2*X1**4)-6*CAI*
20 K0*K20*EXP(K20/(K10*X1))*X2/(K10*X1**4)+K0*K20**3*EXP(K20/(K10*
21 X1))*X2/(K10**3*X1**5)-6*CAI*K0*K20**2*EXP(K20/(K10*X1))*X2/(K1
22 0**2*X1**5)-CAI*K0*K20**3*EXP(K20/(K10*X1))*X2/(K10**3*X1**6)-K
23 0*K20**2*EXP(K20/(K10*X1))/(K10**2*X1**3)+2*CAI*K0*K20*EXP(K20/
24 (K10*X1))/(K10*X1**3)+CAI*K0*K20**2*EXP(K20/(K10*X1))/(K10**2*X
25 1**4))*{(K0*(CAI-X1)-K10*X1**2*EXP(-B/(R*X3)))+{-K0*K20**2*EXP(K
26 20/(K10*X1))*X2/(K10**2*X1**3)+2*CAI*K0*K20*EXP(K20/(K10*X1))*X
27 2/(K10*X1**3)+CAI*K0*K20**2*EXP(K20/(K10*X1))*X2/(K10**2*X1**4)
28 +K0*K20*EXP(K20/(K10*X1))/(K10*X1)-CAI*K0*K20*EXP(K20/(K10*X1))
29 /(K10*X1**2)-K0*EXP(K20/(K10*X1))*{-2*K10*X1*EXP(-E/(R*X3))-K0
30 )+2*K10*X1*(K0*K20*EXP(K20/(K10*X1))/(K10*X1)-CAI*K0*K20*EXP(K2
31 0/(K10*X1))/(K10*X1**2)-K0*EXP(K20/(K10*X1))*EXP(-E/(R*X3)))-(-
32 K20*X2*EXP(-E/(R*X3))+K10*X1**2*EXP(-E/(R*X3))-K0*X2)*{(K0*K2
33 0**2*EXP(K20/(K10*X1))/(K10**2*X1**3)+2*CAI*K0*K20*EXP(K20/(K10
34 *X1))/(K10*X1**3)+CAI*K0*K20**2*EXP(K20/(K10*X1))/(K10**2*X1**4
35 )*(K0*(CAI-X1)-K10*X1**2*EXP(-E/(R*X3)))+(K0*K20*EXP(K20/(K10*
36 X1))/(K10*X1)-CAI*K0*K20*EXP(K20/(K10*X1))/(K10*X1**2)-K0*EXP(K
37 20/(K10*X1))*{-K20*EXP(-E/(R*X3))-K0)-A1*(K0*K20*EXP(K20/(K10
38 *X1))*X2/(K10*X1)-CAI*K0*K20*EXP(K20/(K10*X1))*X2/(K10*X1**2)-K
39 0*EXP(K20/(K10*X1))*X2-K0*X1*EXP(K20/(K10*X1))+CAI*K0*EXP(K20/(
40 K10*X1))-A0*EXP(K20/(K10*X1))*X2+X1*v1-B11*K20/K10)/(K0*K20
41 *EXP(K20/(K10*X1))/(K10*X1)-CAI*K0*K20*EXP(K20/(K10*X1))/(K10*X
42 1**2)-K0*EXP(K20/(K10*X1))*{(E*K10*X1**2*EXP(-B/(R*X3)))/(R*X3**
43 2)-E*K20*X2*EXP(-E/(R*X3))/(R*X3**2))-E*K10*X1**2*(-K0*K20**2*E
44 XP(K20/(K10*X1))*X2/(K10**2*X1**3)+2*CAI*K0*K20*EXP(K20/(K10*X1
45 ))*X2/(K10*X1**3)+CAI*K0*K20**2*EXP(K20/(K10*X1))*X2/(K10**2*X1
46 **4)+K0*K20*EXP(K20/(K10*X1))/(K10*X1)-CAI*K0*K20*EXP(K20/(K10*
47 X1))/(K10*X1**2)-K0*EXP(K20/(K10*X1))*EXP(-E/(R*X3)))/(R*X3**2)
    
```